European COST F3 Conference on System Identification & Structural Health Monitoring, Madrid, Spain, June 6-9, 2000.

STATISTICAL PROCESS CONTROL AND PROJECTION TECHNIQUES FOR STRUCTURAL HEALTH MONITORING

Hoon Sohn[1] and Charles R. Farrar[2]

[1,2] Engineering Analysis Group, MS P-946, Los Alamos National Laboratory, Los Alamos, New Mexico, 87545, USA

SUMMARY: This paper focuses on applying statistical process control techniques to vibration-based damage diagnosis. First, an auto-regressive (AR) model is fit to the measured acceleration-time histories from an undamaged structure. Coefficients of the AR model are selected as the damage-sensitive features for the subsequent control chart analysis. Finally, the AR coefficients of the models fit to subsequent new data are monitored relative to the control limits. A unique aspect of this study is the coupling of various projection techniques such as principal component analysis, linear and quadratic discriminant operators with the statistical process control in an effort to enhance the discrimination between features from the undamaged and damaged structures. This combined statistical procedure is applied to vibration test data acquired from a concrete bridge column as the column is progressively damaged. The coupled approach captures a clearer distinction between undamaged and damaged vibration responses.

KEYWORDS: Damage Detection, Statistical Process Control, Pattern Recognition, Concrete Bride Column, Auto-Regressive Model, Control Chart, Unsupervised Learning

INTRODUCTION

Structural health-monitoring, reviewed in detail in [1], is best studied in the context of a statistical pattern recognition paradigm. This paradigm can be described as a four-part process: 1.) Operational evaluation, 2.) Data acquisition & cleansing, 3.) Feature extraction & data reduction, and 4.) Statistical model development. In particular, this paper focuses on Parts 3 and 4 of the process. The process is illustrated through application to acceleration-time history data measured on an undamaged and subsequently damaged concrete columns. Note that the primary objective of this study is to identify the existence of damage. The localization and quantification of damage are not addressed in this study.

In this study auto-regressive (AR) models are fit to measured time series. The coefficients of these models are the damage-sensitive features that are subsequently used to discriminate between a damaged and undamaged structure. Several projection techniques such as principal component analysis (PCA) and linear/quadratic projections [2] are applied to the data acquired in this study in an effort to reduce the dimensionality of the data and enhance the discrimination between features from undamaged and damaged structures. Statistical model development is concerned with the implementation of the algorithms that analyze the distribution of extracted features in an effort to determine the damage state of the structure. In this study techniques from statistical process control are applied to the selected features to investigate the existence of damage in the structure of interest.

SPATIAL DATA COMPRESSION

In this study, PCA is used to perform data compression prior to the feature extraction process when data from multiple measurement points are available. This process transforms the time series from multiple measurement points into a single time series by forming a linear combination of the various measured time histories. Inherent in any data compression process is the loss of information. Principal component analysis is employed in this study to preserve as much of relevant information as possible during the dimensionality reduction.

Consider the response parameter time histories, $u_i(t_j)$, corresponding to *m* different measurement degrees of freedom and sampled at *n* time intervals

$$\begin{array}{cccc} u_1(t_1) & u_1(t_2) & \cdots & u_1(t_n) \\ u_2(t_1) & u_2(t_2) & \cdots & u_2(t_n) \\ \vdots & \vdots & & \vdots \\ u_m(t_1) & u_m(t_2) & \cdots & u_m(t_n) \end{array} \tag{1}$$

The time histories are first normalized by subtracting their respective mean values, $\bar{u}_i$, defined as.

$$\bar{u}_i = \frac{1}{n}\sum_{j=1}^{n} u_i(t_j) \tag{2}$$

At a given time, t_j, a vector of the the normalized response components corresponding to the *m* measurement degrees of freedom is formed as

$$\mathbf{u}(t_j) = \left[\, u_1(t_j) - \bar{u}_1 \quad u_2(t_j) - \bar{u}_2 \cdots u_m(t_j) - \bar{u}_m \,\right]^T \tag{3}$$

Then, the $m \times m$ covariance matrix, Ω, among spatial measurement locations summed over all time samples is given by

$$\Omega = \sum_{j=1}^{n} \mathbf{u}(t_j)\,\mathbf{u}(t_j)^T \tag{4}$$

The eigenvalues, λ_i, and eigenvectors, $\mathbf{v}_i$, of the covariance matrix satisfy:

$$\Omega\,\mathbf{v}_i = \lambda_i \mathbf{v}_i \tag{5}$$

Here, an eigenvector $\mathbf{v}_i$ is also called a *principal component*. To reduce the *m*-dimensional vector $\mathbf{u}(t)$ into a *d*-dimensional vector, $\mathbf{x}_v(t)$, where $d < m$, $\mathbf{u}(t)$ is projected onto the eigenvectors corresponding to first *d* largest eigenvalues:

$$\mathbf{x}_v(t) = [\mathbf{v}_1 \quad \cdots \quad \mathbf{v}_d]^T \mathbf{u}(t) \tag{6}$$

FEATURE EXTRACTION

In this study the coefficients of an AR model are selected as damage sensitive features. The time series from an individual measurement point, or the spatially-compressed time series obtained from PCA, can be used to construct the AR models. In the AR model the current point in a time series is modeled as a linear combination of the previous *n* points:

$$y(t) = \sum_{j=1}^{n} \phi_j \ y(t-j) \ + e(t) \ , \tag{7}$$

where y(t) is the time history at time t, ϕ_j is an unknown auto-regression coefficient, and e(t) is an unobservable random error with zero mean and constant variance. For the application reported herein, the time signals are divided into smaller size time windows, and AR coefficients are estimated from each time window. Following this procedure, a large set of AR coefficients are obtained for subsequent damage diagnoses.

DATA COMPRESSION OF FEATURE VECTORS: LINEAR AND QUADRATIC PROJECTIONS

In this study the multi-dimensional feature vectors are projected onto one-dimensional subspaces using linear and quadratic projections [3,4], and the statistical discrimination procedure is applied to the one-dimensional variable. The specific linear and quadratic projections, are derived by considering the Bayesian classification problem. The classification problem consists of determining which class an observation belongs to given a discriminant function. To illustrate this, consider a situation in which there are only two classes (classes A and B) and multi-dimensional feature vector **x** is obtained. The first task is to determine a rule, which can be used to assign the data to one of the two classes. A decision rule based on Bayes' Theorem is shown to minimize the probability of error, which is the probability of misclassification of assigning a new feature to class A when, in fact, it belong to class B, or vice versa [4]. If the product of the conditional probability and the prior probability replaces the corresponding posterior probability and the conditional densities are normal distributions, the following quadratic decision rule can be obtained:

$$D(\mathbf{x}) = \ln(d(\mathbf{x})) = -\frac{1}{2}(\mathbf{x}-\mathbf{m}_A)^T \mathbf{\Sigma}_A^{-1}(\mathbf{x}-\mathbf{m}_A) + \frac{1}{2}(\mathbf{x}-\mathbf{m}_B)^T \mathbf{\Sigma}_B^{-1}(\mathbf{x}-\mathbf{m}_B) + \frac{1}{2}\ln\frac{|\mathbf{\Sigma}_B|}{|\mathbf{\Sigma}_A|} \tag{8}$$

where $\mathbf{m}_A$ and $\mathbf{m}_B$ are the mean vectors of the classes A and B distributions, respectively. $\mathbf{\Sigma}_A$ and $\mathbf{\Sigma}_B$ are the covariance matrices of each class. Now the feature **x** is assigned to class A when $D(\mathbf{x}) > 0$. Otherwise **x** is assigned to class B. Note that the prior probabilities $P(A)$ and $P(B)$ are assumed to be identical in Equation (8) for simplicity. Otherwise the feature **x** is assigned to class A when $D(\mathbf{x}) > \ln(P(B)/P(A))$. In the case where the covariance matrices are identical matrices ($\mathbf{\Sigma} = \mathbf{\Sigma}_A = \mathbf{\Sigma}_B$), the classification boundary can be further simplified to a linear form:

$$D(\mathbf{x}) = (\mathbf{m}_A - \mathbf{m}_B)^T \mathbf{\Sigma}^{-1}\mathbf{x} + \frac{1}{2}(\mathbf{m}_B^T \mathbf{\Sigma}^{-1}\mathbf{m}_B - \mathbf{m}_A^T \mathbf{\Sigma}^{-1}\mathbf{m}_A) \tag{9}$$

Although the Bayesian classifier minimizes the probability of error, the Bayesian classifier requires the conditional probability densities for each class to successfully implement such a classification scheme. To overcome this difficulty, the decision rule $\mathrm{D}(\mathbf{x})$ in Equations (8) and (9) can be rewritten in the more general forms. For a quadratic discriminant function,

$$\mathrm{D}(\mathbf{x}) = \mathbf{x}^{\mathrm{T}}\mathbf{Q}\,\mathbf{x} + \mathbf{V}\mathbf{x} \tag{10}$$

and for a linear discriminant function:

$$\mathrm{D}(\mathbf{x}) = \mathbf{F}^{\mathrm{T}}\mathbf{x} \tag{11}$$

Note that the constant terms in Equations (8) and (9) are omitted in Equations (10) and (11) without losing generality. The decision rule can be also viewed as a projection that maps multi-dimensional space $\mathbf{x}$ to a one-dimensional space $\mathrm{D}(\mathbf{x})$. We are particularly interested in defining a transformed feature $\tau = \mathrm{D}(\mathbf{x})$ such that the means of two classes are as far as possible and their variances are the smallest as possible after either quadratic or linear projections. These projections can be sought by maximize the following Fisher criterion [3]:

$$f = \frac{(\mathrm{m}_{\mathrm{A}} - \mathrm{m}_{\mathrm{B}})^2}{\sigma_{\mathrm{A}}^2 - \sigma_{\mathrm{B}}^2} \tag{12}$$

where m_{A} and m_{B} are the means of the projected feature in classes A and B. σ_{A} and σ_{B} are the corresponding standard deviations of the transformed features. For a linear projection in Equation (11), the quantities, m_{A}, m_{B}, σ_{A}, and σ_{B} linearly depend on $\mathbf{F}$. Therefore, the Fisher criterion can be expressed as a function of $\mathbf{F}$. Taking derivatives of f with respect to $\mathbf{F}$ and setting this quantity equal to zero yields the following linear projection [2]:

$$\mathbf{F} = 2(\mathbf{\Sigma}_{\mathrm{A}} + \mathbf{\Sigma}_{\mathrm{B}})^{-1}(\mathbf{m}_{\mathrm{A}} - \mathbf{m}_{\mathrm{B}}) \tag{13}$$

It is important to mention that the performance of the linear classifier will not be optimal unless $\mathbf{\Sigma}_{\mathrm{A}}$ and $\mathbf{\Sigma}_{\mathrm{B}}$ are the same. For the test data studied herein, acceleration data from undamaged and damaged classed are observed to have unequal covariance matrices. Because the Bayesian decision boundary is quadratic under the more general circumstance of unequal covariance matrices between classes, the quadratic transformation yields the best discrimination power. Introducing a new variable y_i, which represents the product of two x_i's, Equation (10) can be linearized as [4]:

$$\mathrm{D}(\mathbf{x}) = \sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}x_i x_j + \sum_{i=1}^{n} v_i x_i = \sum_{i=1}^{n(n+1)/2} a_i y_i + \sum_{i=1}^{n} v_i x_i \tag{14}$$

where q_{ij}, v_i are the components of $\mathbf{Q}$ and $\mathbf{V}$ respectively. y_i represents the product of the x_j's and a_i is the corresponding entry in the $\mathbf{Q}$ matrix. In addition, n is the order of the AR model or the dimension of AR coefficients defined in Equation (7). Let $\mathbf{Y}$ and $\mathbf{X}$ denote column vectors of y_i's and x_j's, respectively. Now, the following equation analogous to the linear case can be solved for $\mathbf{Q}$ and $\mathbf{V}$ by introducing a new variables vector $\mathbf{Z} = [\,\mathbf{Y}^{\mathrm{T}}\ \ \mathbf{X}^{\mathrm{T}}\,]^{\mathrm{T}}$ and letting $\mathbf{E}$ and $\mathbf{S}$ be the expected vector and covariance matrix of $\mathbf{Z}$, respectively:

$$[a_1 \cdots\ a_{n(n+1)/2}\ v_1\ \cdots\ v_n]^{\mathrm{T}} = 2[\mathbf{S}_{\mathrm{A}} + \mathbf{S}_{\mathrm{B}}]^{-1}(\mathbf{E}_{\mathrm{A}} - \mathbf{E}_{\mathrm{B}}) \tag{15}$$

Then a_i's and v_j's can be rearranged to form the $\mathbf{Q}$ matrix and $\mathbf{V}$ vector. Note that the projection techniques presented here are used for a dimensionality reduction purpose rather than for a construction of a discriminant function. That is, the n-dimensional AR coefficient space is projected onto a single scalar space maximizing the mean differences between two classes. Damage diagnosis is conducted on the transformed feature using the statistical process control technique described in the following section.

STATISTICAL MODELING: STATISTICAL PROCESS CONTROL

Statistical process control (SPC) is a collection of tools useful in process monitoring, and improvement. The control chart is the most commonly used one and very suitable for automated continuous system monitoring [5]. When the system of interest experiences abnormal conditions, the mean and/or variance of the extracted features are expected to change. In this study an X-bar control chart is employed to monitor the changes of the selected feature means and to identify samples that are inconsistent with the past data sets. Several variations of the control charts can be found in Reference [5]. To monitor the mean variation of the features, the features are first arranged in subgroups of size *p* where τ_{ij} is the extracted feature from previous section and *q* is the number of subgroups. The subgroup size *p* is often taken to be 4 or 5 [5]. If *p* is chosen too large, a drift present in individual subgroup mean may be obscured, or averaged-out. An additional motivation for the using subgroups, as opposed to individual observations, is that the distribution of the subgroup mean values can be reasonably approximated by a normal distribution as a result of central limit theorem. Next, the subgroup mean $\overline{\mathrm{X}}_i$ and standard deviation S_i of the features are computed for each subgroup ($i = 1, \cdots, q$):

$$\overline{\mathrm{X}}_i = mean\left(\tau_{ij}\right) \text{ and } \mathrm{S}_i = std(\tau_{ij}) \tag{16}$$

Here, the mean and standard deviation are with respect to *p* observations in each subgroup. Finally, an X-bar control chart is constructed by drawing a centerline (CL) at the subgroup mean and two additional horizontal lines corresponding to the upper and lower control limits (UCL & LCL) versus subgroup numbers (or with respect to time). The centerline and two control limits are defined as follows:

$$\mathrm{CL} = mean(\overline{\mathrm{X}}_i)\,,\ \mathrm{UCL} = \mathrm{CL} + Z_{\alpha/2}\frac{\mathrm{S}}{\sqrt{n}}\,,\text{ and } \mathrm{LCL} = \mathrm{CL} - Z_{\alpha/2}\frac{\mathrm{S}}{\sqrt{n}} \tag{17}$$

where the calculation of mean is with respect to all subgroups ($i = 1, \cdots, q$). $Z_{\alpha/2}$ is the percentage point of the normal distribution with zero mean and unit variance such that $P[z \geq Z_{\alpha/2}] = \alpha/2$. The variance S^2 is estimated by averaging the variance S_i of all subgroups. Note that, if $\overline{\mathrm{X}}_\mathrm{i}$ can be approximated by a normal distribution due to the central limit theorem, the control limits in Equation (17) correspond to a $100(1-\alpha)\,\%$ confidence interval. If the system experienced damage, this would likely be indicated by an unusual number of subgroup means outside the control limits; a charted value outside the control limits is referred to as an *outlier* in this paper. The monitoring of damage occurrence is performed by plotting new $\overline{\mathrm{X}}_i$ values relative to along the previous control limits.

In general, the observation of a large number of outliers does not necessary indicate that the structure is damaged but only that system has varied to cause statistically significant changes in its vibration signatures. This variability can be caused by a variety of environmental and operational conditions that the system is subject to. Because the influence of operational and environmental factors on the dynamic characteristics of the test structure was minimal for the presented laboratory test, the deterioration of the structure was assumed to be the main cause of the abnormal changes of the system.

APPLICATION TO CONCRETE COLUMNS

Faculty, students and staff at the University of California, Irvine (UCI) performed quasi-static, cyclic tests to failure on seismically retrofitted, reinforced-concrete bridge columns. Vibration tests were performed on the columns at intermittent stages during the static load cycle testing when various amounts of damage had been accumulated in the columns. The configuration and dimension of the test column the dimensions are shown in Fig. 1. Details of testing and all test data can be found at http://ext.lanl.gov/projects/damage_id/.

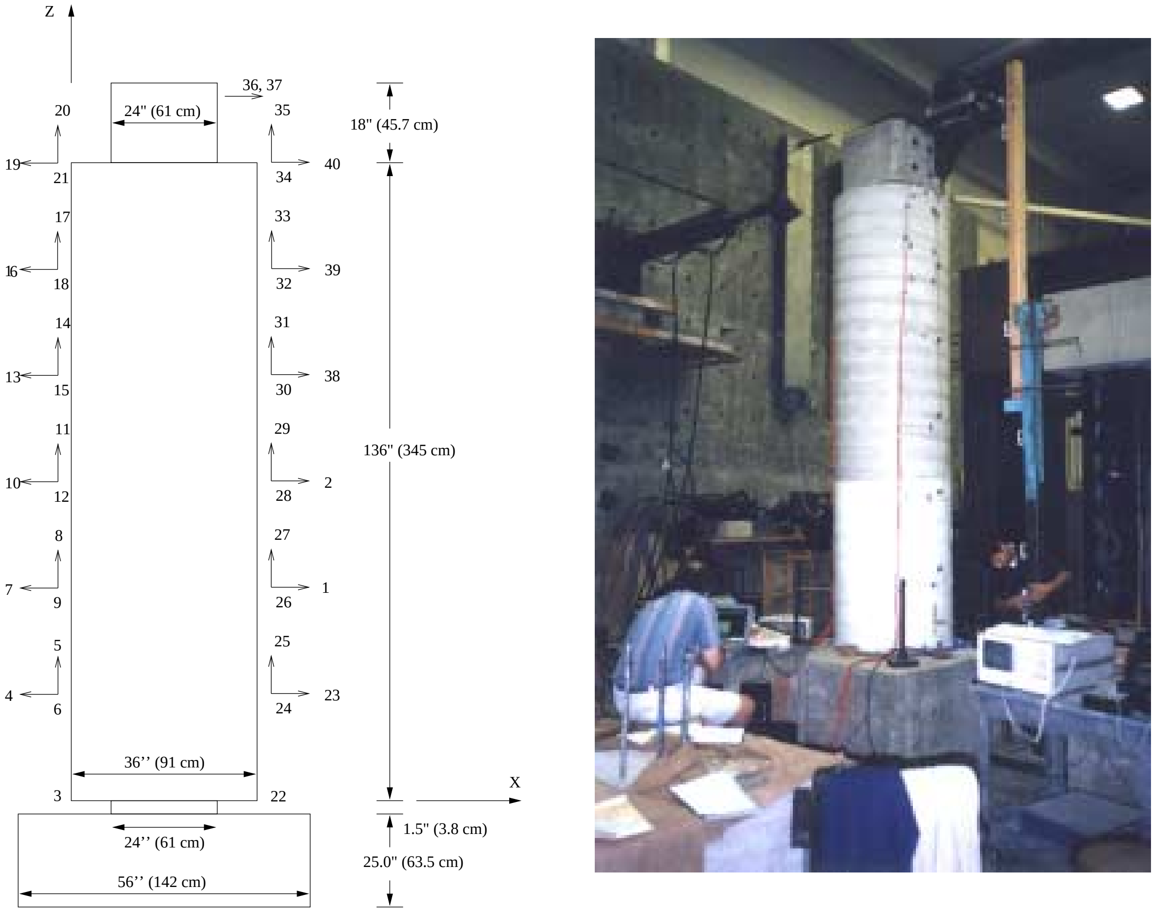

Figure 1: Column dimensions and photo of an actual test structure.

Feature Extraction and Data Compression

The applicability of SPC to damage diagnosis problem is demonstrated using individual time series from different measurement points. The 8192 point measured time series are first divided into 512 16-point subgroups, and a third-order AR model is fit to individual subgroups resulting in 512 sets of AR coefficients. Damage diagnoses using X-bar control charts are performed using the first AR coefficient as a feature. The control limits corresponding to a 99% confidence interval are constructed by setting $\alpha = 0.01$ in Equation (17). Next, the advantage of projection techniques is investigated. Linear and quadratic projections are introduced to map multi-dimensional AR coefficient space into a one-dimensional space in order to maximize the mean differences between the data sets obtained from the undamaged and damaged classes. SPC analyses are then conducted on the transformed single scale feature. Finally, PCA is carried out to all response time series for spatial dimensionality reduction prior to feature selection and SPC analysis. That is, all time series from 39 response points are projected onto the first principal component of the covariance matrix of the time series. The subsequent feature selection and SPC analyses are performed based on this single time series that is a linear combination of the 39 measured time series.

Statistical Modeling: X-bar Control Chart using the First AR Coefficient

Figure 1 shows the damage diagnosis results using the first coefficient of the third order AR model. These results here and in all subsequent plots are shown for the undamaged data as well as data from the first, second and final (fifth) damage levels. Time histories from measurement point 1 are used for the construction of the control chart. UCL, LCL, and CL denote the upper and lower control limits, and centerline obtained from the time series of the undamaged structure. Note that the extracted feature is standardized prior to the construction of the control chart by subtracting the mean and dividing by the standard deviation. After establishing the control limits and centerline, features obtained at each damage level are plotted relative to the control limits and centerline obtained from the undamaged data. The outliers, which are samples outside the control limits, are marked by "+" sign in all figures. Note that the subgroup mean and standard deviation obtained from damage level 0 are used to normalize all the subsequent damage levels.

The diagnosis results using the other AR coefficients are also summarized in Table 1. For this particular example, the third AR coefficient seems to be most indicative of damage, and the first coefficient is very insensitive to damage. For damage levels 0 and 1, the numbers of total outliers out of 384 samples are 2 and 1, respectively. Therefore, it is not clear if the system experienced any significant damage at damage level 1 based on the analysis of the X-bar control chart using the individual AR coefficients.

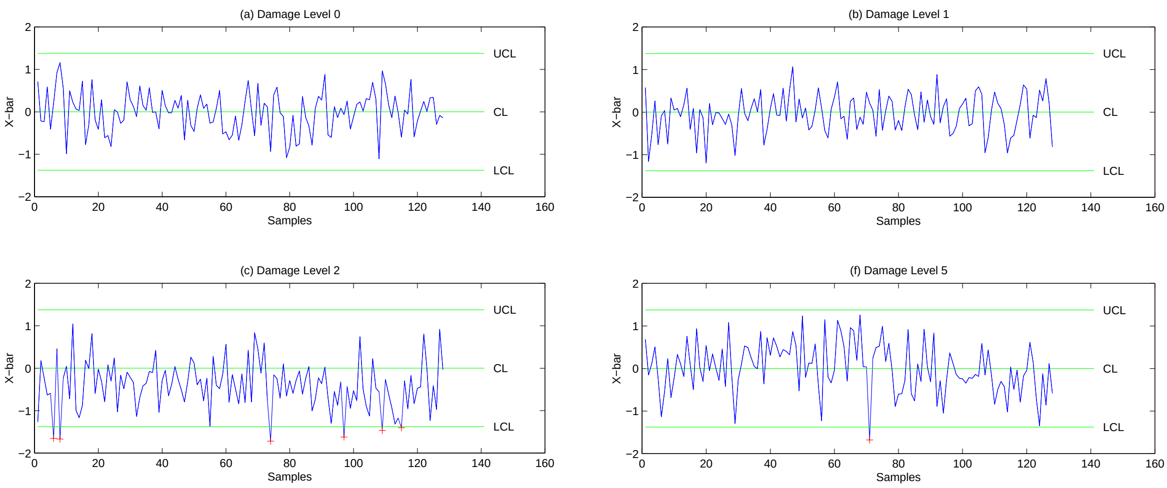

Figure 1: X-bar control chart using the first AR coefficient

Table 1: Outlier numbers of X-bar control chart using different AR coefficients

AR coefficient	Damage Level					
	0	1	2	3	4	5
α_1	0/128*	0/128	6/128	6/128	2/128	1/128
α_2	0/128	0/128	6/128	10/128	30/128	23/128
α_3	2/128	1/128	12/128	31/128	77/128	88/128
Total # of outliers	2/384 (0.52%)	1/384 (0.26%)	24/384 (6.25%)	47/384 (12.24%)	109/384 (28.39%)	112/384 (29.17%)

*1/128 indicates that there is a single outlier out of 128 sample data points.

Statistical Modeling: Control Chart Analysis after Linear or Quadratic Projection

Next, the linear and quadratic projection techniques are incorporated into X-bar control chart. To overcome these difficulties with the construction of multiple control charts using individual AR coefficients, a single control chart is developed using a one-dimensional transformed feature. The three-dimensional AR coefficients are first projected onto one-dimensional space, and the X-bar chart is constructed based on the transformed feature

Figure 2 and Figure 3 show the construction of control charts (from time series at Pt.1) and the process monitoring after linear and quadratic projections, respectively. Comparison of these figures with Figure 1 clearly reveals the improvement of diagnosis performance. Diagnosis results using the other measurement points are conducted and the similar performance improvement is observed. However, the diagnosis results are not presented in this paper because of space limitation. As mentioned earlier, the linear projection is not the optimal projection in the examples presented here because the orders of two class covariance matrices (one from the undamaged case and the other from each damage level) are quite different. In theory, the quadratic projection is the optimal one in a sense of minimizing the error of misclassification. However, no significant performance difference between linear and quadratic projections is observed in these examples (see Table 2).

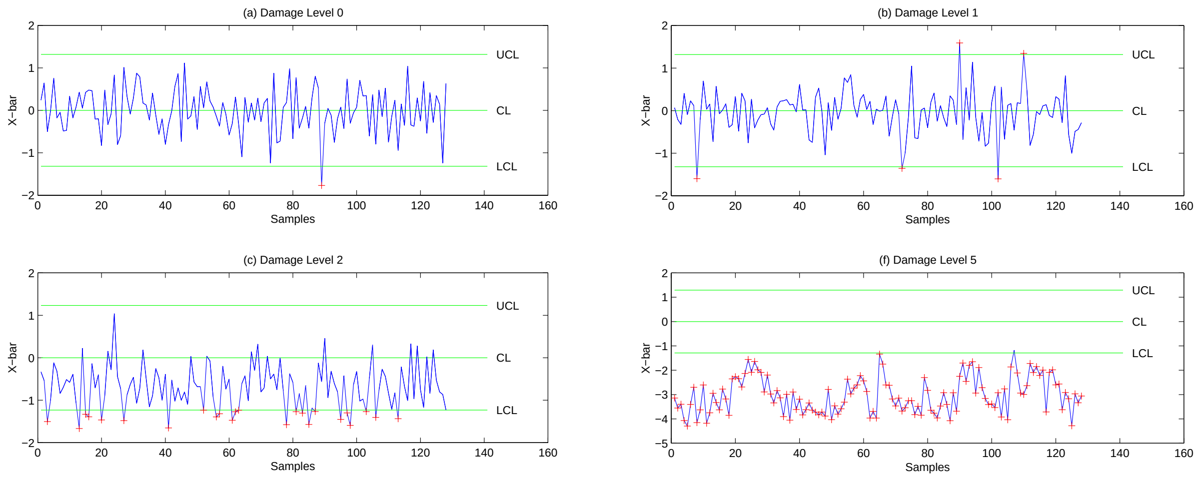

Figure 2: X-bar control chart of the coefficients after linear projection

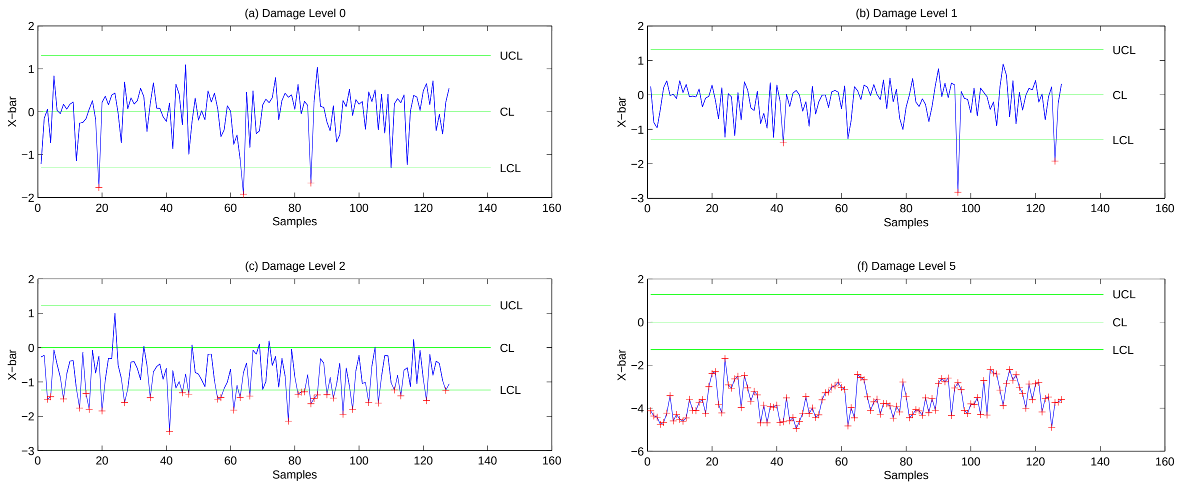

Figure 3: X-bar control chart of the coefficients after quadratic projection

Principal Component Analysis

The PCA conducted on the covariance matrix of 39 response time series indicates that the responses of 39 measurement points are closely correlated. The individual and cumulative eigenvalues of the covariance matrix are shown in Figure 4. Particularly, the first principal component alone holds about 30% of total information and in the following examples, raw time series from the 39 measurement points are first projected onto this first principal

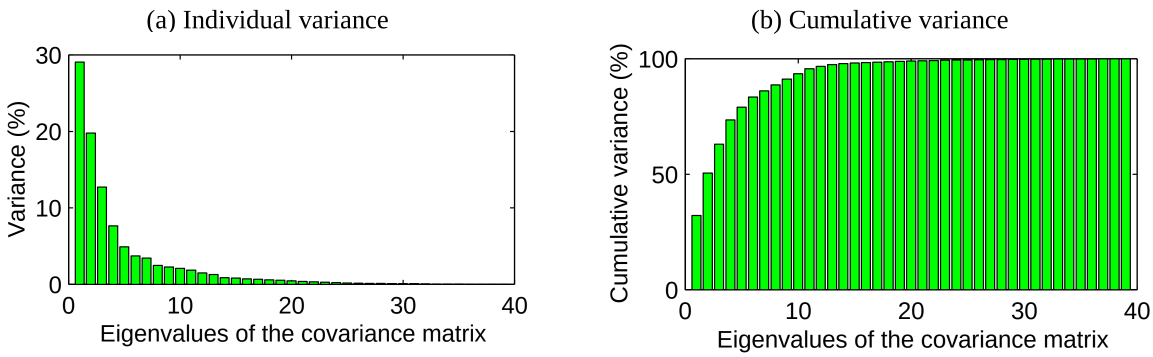

Figure 4: Principal component analysis of the covariance matrix of 39 response points

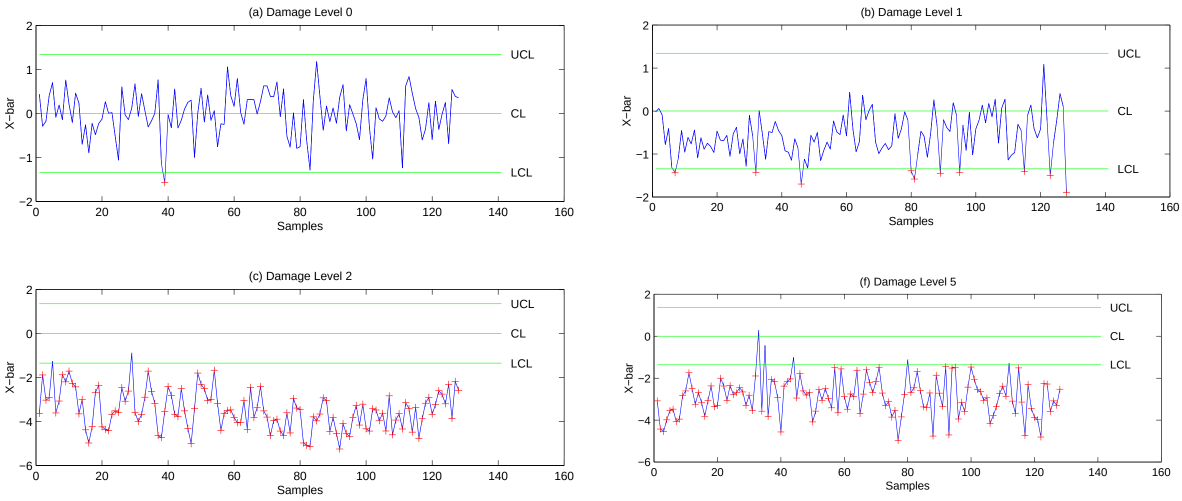

Figure 5: X-bar control chart of the AR coefficients after principal component analysis of all measurement points and linear projection

Table 2: Outlier numbers of X-bar control chart using linear or quadratic projection

Projection	Damage Level					
	0	1	2	3	4	5
Linear	1/128	5/128	24/128	125/128	121/128	127/128
Quadratic	3/128	3/128	34/128	128/128	127/128	128/128

Table 3: Damage diagnosis results after PCA and linear/quadratic projections

Projection	Damage Level					
	0	1	2	3	4	5
Linear	1/128 (0.78%)	10/128 (7.81%)	126/128 (98.44%)	127/128 (99.22%)	121/128 (94.53%)	123/128 (96.09%)
Quadratic	1/128 (0.78%)	7/128 (5.47%)	126/128 (98.44%)	127/128 (99.22%)	121/128 (94.53%)	124/128 (96.88%)

component. The subsequent feature extraction and X-bar control chart analyses are performed in the same fashion as before. Since the linear and quadratic projections have produced similar results, only the damage diagnosis results after PCA and the linear projection are displayed in Figure 5. The results of Table 3 are equivalent to or slightly better than those of Table 2 and much better than those of Table 1.

SUMMARY AND DISCUSSION

The proposed damage detection approach is used to identify the damage evolution in a concrete bridge column based on the data obtained from the vibration tests. AR models are constructed using the measured time signals, and damage diagnoses using X-bar control charts are performed using the first AR coefficient as damage-sensitive features. Next, the advantage of projection techniques is investigated. Linear and quadratic projections are introduced to map the multi-dimensional AR coefficient feature space into a one-dimensional feature space in order to maximize the differences in the mean values between the two data sets being compared. The control chart analysis is then conducted on the transformed single dimension feature data. Finally, PCA is carried out on all response time series for spatial dimensionality reduction prior to feature extraction. That is, all time series from multiple measurement points are projected onto the first principal component of the time series covariance matrix, and the subsequent feature selection is performed based on this compressed time series data.

The projection techniques improved the performance of control chart analysis compared to the damage diagnosis using the individual AR coefficients. When the projection techniques and PCA are combined, the control charts successfully indicated the system response anomaly for all damage levels by showing a statistically significant number of outliers outside the control limits. It should be also noted that this study is carried out in an unsupervised learning mode. Although the projection techniques require two separate data sets, it is only assumed that there is one data set from the undamaged class and the other data set is from an *unknown* class. Unsupervised learning techniques are very important for civil engineering studies because response data from a similar damaged system is rarely available.

ACKNOWLEDGEMENT

This work was funded by the Dept. of Energy's ASCI program and a cooperative research agreement with Kinematrics Corp. Column vibration tests were conducted jointly with Prof. Pardoen at the Univ. of California, Irvine under CALTRANS funding.

REFERENCES

1. Doebling, S. W., Farrar, C. R., Prime, M. B. and Shevitz, D. W., "A Review of Damage Identification Methods that Examine Changes in Dynamic Properties," *Shock and Vibration Digest* **30** (2), pp. 91-105, 1998.

2. Bishop, C. M., *Neural Networks for Pattern Recognition*, Oxford University Press, 1995.

3. Fisher, R. A., *The Mathematical Theory of Probabilities*, Macmillan, New York, 1923.

4. Fukunaga, K., *Introduction to Statistical Pattern Recognition*, Academic Press, NY,1990.

5. Montgomery, D. C., *Introduction to Statistical Quality Control*, John Wiley & Sons,1997.